region of New Mexico; and at San Cristobal in still another part of the state, N. C. Nelson has excavated a stratified deposit showing four successive layers of different type. It is quite likely that some of the types at these three sites will prove to be similar, or even identical, as soon as the material can be compared. In this event a chronological frame work would be established that may prove capable of extension to accommodate a considerable part of the prehistoric data from the Southwest, and to fix distinctive and otherwise undatable local variations of ancient culture. The impression that there were at least two principal periods in the Southwest, the earlier represented by what are currently called Cliff Dweller forms, has of course long been prevalent, but the supporting evidence has been random. The three present sequential determinations promise not only definitely to establish but to elaborate the older general conviction.

The findings here discussed will be published in the Anthropological Papers of the American Museum of Natural History.

THE NUMERICAL RESULTS OF DIVERSE SYSTEMS OF BREEDING

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When organisms differing with respect to a pair of characters are bred for generation after generation, the relative numbers of individuals that will show any particular combination of characters in any given generation of course depend on the system of mating followed. The different classes of individuals may mate at random; or there may be assortative mating (dominants with dominants, recessives with recessives); or dominants alone may be bred; or recessives alone. Or again, self fertilization may prevail; or one of the various possible types of inbreeding may be followed. If we represent the two alternative characters by A (dominant) and a (recessive), then three types of individuals are possible, AA, Aa, and aa. The question here raised is as to the relative numbers of each of these three types of individuals after any number n of generations of mating, by any of the systems mentioned above. This depends of course on the constitution of the parents at the beginning, as well as on the system of breeding and the number of generations.

When breeding by a given system is continued for many generations, several types of results may be distinguished:

A. In some cases the proportions of the population having particular

constitutions (AA, Aa, or aa) remain the same for all generations. Examples are:

(1) Random mating when the parents are at the beginning either all Aa; or half AA, half aa; or when the parental proportions are AA+2Aa +aa. In these cases after any number of generations the resulting progeny show the proportions AA+2Aa+aa.

(2) Random mating, parental proportions, rAA to $t$aa. Constant proportion in offspring: r^2AA+$2rt$Aa+t^2aa.

(3) Random mating, parental proportions, AA+Aa. Constant result, 9AA+6Aa+aa.

(4) Random mating, parental proportions, rAA+sAa. Constant result, $(s+2r)^2$AA+$2s(s+2r)$Aa+s^2aa.

(5) Inbreeding, daughters bred back to father: both parents Aa. Constant result, AA+2Aa+aa.

(6) Sex-linked factors; random mating; original parents rAA+rA$-$+ $t$aa+ta$-$, in equal numbers. Constant later proportions: males, rA$-$ + ta$-$; females, r^2AA+$2rt$Aa+t^2aa. Males and females in equal numbers.

B. In some complex cases the population changes from generation, and the best we can do is to obtain a formula which shall, when we know the proportions in a given generation n, give us the proportions in the next following generation $n+1$.

Examples:

(7) Random mating, population in nth generation: rAA+sAa+$t$aa. Population in generation $n+1 = (s+2r)^2$AA + $2(s+2r)(s+2t)$Aa + $(s+2t)^2$aa.

(8) Assortative mating; population in the generation n is rAA+ sAa+$t$aa. In the next generation $(n+1)$ the population is $(2r+s)^2$AA+ $(4rs+2s^2)$Aa+$(s^2+4rt+4st)$aa.

(9) Selection of dominants; parental population in generation n is rAA+sAa+$t$aa. Population in the next generation $(n+1)$ is $(2r+s)^2$ AA+$2s(2r+s)$Aa+s^2aa.

C. In many cases the constitution of the population changes from generation to generation, as long as the given system of breeding lasts, and it is possible to give a simple formula in terms of n, the number of generations that the given system has been followed, from which formula the proportions in generation n can be directly computed.

Examples:

(10) Assortative mating (dominants with dominants, recessives with recessives); original parents are progeny of the cross AA $\times$ aa, so that their proportions are AA+2Aa+aa. After n assortative matings the proportions are: $(n+1)$AA+2Aa+$(n+1)$aa.

(11) Assortative mating; parental population is the result of the random mating of rAA+$t$aa. After n assortative matings the population is $r(r+nt)$AA+$2rt$Aa+$(n+1)t^2$aa.

(12) Assortative mating; parental population all Aa. After n assortative matings the population is nAA+2Aa+$n$aa.

(13) Assortative mating; parental population AA and Aa in equal numbers. After n matings the population is $(3n+6)$AA+6Aa+$n$aa.

(14) Selection of dominants; parental population AA+2Aa+aa. After n generations the population is $(n+1)^2$AA + $2(n+1)$Aa + aa.

(15) Self-fertilization; parents all Aa. After n self-fertilizations the population is (2^n-1)AA + 2Aa + (2^n-1)aa.

(16) Self-fertilization; population at the beginning, rAA + sAa + $t$aa. After n generations the population is $[r(2^{n+1})+s(2^n-1)]$AA + $2s$Aa + $[t(2^{n+1})+s(2^n-1)]$aa.

D. In most cases the constitution of the population changes from generation to generation, giving a series of values not readily expressed in terms of n (the number of generations) alone. In these cases diverse systems or diverse parents give different series of values, almost all of which are examples of certain simply derived mathematical series, or of their combinations. The results are therefore best presented by giving first these fundamental series, each with its designation. Then the results of any number of generations of any system of breeding can be given by designation of the series which it forms. The main series, with their first 10 terms, are given in the following table:

Table of the Fundamental Series in Mendelian Breeding

SERIES	HOW FORMED												
n	$x+1$	0[1]	1	2	3	4	5	6	7	8	9	10	
B	2x	1	2	4	8	16	32	64	128	256	512	1024	$(=2^n)$
C	2x+1	0	1	3	7	15	31	63	127	255	511	1023	$(=2^n-1)$
D	2x−1		2	3	5	9	17	33	65	129	257	513	$(=2^{n-1}+1)$
E	2x+1			2	5	11	23	47	95	191	383	767	$(=2^n-2^{n-2}-1)$
F	Sum of two preceding	0	1	1	2	3	5	8	13	21	34	55	
G	2x+1, then 2x−1	0	1	1	3	5	11	21	43	85	171	341	$(=B_{n-1}-G_{n-1})$
H	G−F	0	0	0	1	2	6	13	30	64	137	286	
I	B−G−F	1	[illegible]	2	3	8	16	35	72	150	307	628	$(=G_{n+1}-F_n)$
J	B_n-F_{n+1}	0	1	2	5	11	24	51	107	222	457	935	
K	B_n-F_{n+2}	0	0	1	3	8	19	43	94	201	423	880	
L	$B_n-F_{n-1}-G_{n-1}$		2	2	6	11	24	48	99	200	406	819	
M	$2x+F_{n-1}$	2	4	9	19	40	83	171	350	713	1447	2952	$(=3B_n-F_{n+2})$

The table gives in the first column a letter by which the series will be designated (as n, B, C, J, etc). In the next column is given the method by which any given term of the series is formed from the preceding term, this preceding term being designated x. Thus, under C, the expression, $2x+1$ means that to obtain any term of the series, we double the preceding term and add 1. In some of the later series the derivation here given shows that the given series is formed by some combination of two previous series. Thus, series H is formed by substracting from any term of G the corresponding term of F. Any given term of a series is designated by a subscript representing the corresponding term of the series n: thus, F_4 means the fourth term of the series F (it is therefore 3); J_n means the nth term of the series J; D_{n+1} means the $(n+1)$th term of D; etc. At the end of each series is given certain other information; showing as a rule how any term of the series may be otherwise designated; or another method of derivation.

The results of any system of breeding that falls in the present group may be given by specifying which series of the table it gives and what terms of the series correspond to a given generation. Examples will make this clear:

(17) Self-fertilization, beginning with a cross between AA and Aa. After n self-fertilizations the proportions of the resulting population are given by the following:

$$AA = \frac{E_{n+2}}{B_{n+2}}. \qquad Aa = \frac{1}{B_{n+1}}. \qquad aa = \frac{C_n}{B_{n+2}}.$$

This signifies that for any generation n, the proportion of AA will be found by dividing the term $(n+2)$ of the series E by the term $(n+2)$ of the series B. So, after one self-fertilization the proportion that are AA will be 5/8; after two, 11/16; after eight, 767/1024. After five generations the proportion that are Aa will be 1/64; of aa, 31/128; and so on.

(18) Self-fertilization in a typical Mendelian population composed of AA+2Aa+aa. After n generations:

$$AA = \frac{C_{n+1}}{B_{n+2}}. \qquad Aa = \frac{1}{B_{n+1}}. \qquad aa = \frac{C_{n+1}}{B_{n+2}}.$$

(19) Inbreeding, brother by sister, in the progeny derived from a cross of AA by aa. After n generations:

$$AA = \frac{J_n}{B_{n+1}}. \qquad Aa = \frac{F_{n+1}}{B_n}. \qquad aa = \frac{J_n}{B_{n+1}}.$$

(20) Inbreeding, brother by sister, when the original cross was AA by Aa. After *n* inbreedings:

$$AA = \frac{M_{n+1}}{B_{n+3}}. \qquad Aa = \frac{F_{n+3}}{B_{n+2}}. \qquad aa = \frac{K_{n+1}}{B_{n+3}}.$$

(21) Inbreeding, parent by offspring; parents AA by Aa; progeny mated back continually to Aa:

$$AA = \frac{D_{n+1}}{B_{n+2}}. \qquad Aa = \frac{1}{2}. \qquad aa = \frac{C_n}{B_{n+2}}.$$

(22) Sex-linked characters; a cross between AA and a−, then breeding by random mating. After *n* random matings:

$$\text{Males:}\quad A- = \frac{G_{n+1}}{B_n}\text{: thus, 1/2, 3/4, 5/8, etc.}$$

$$a- = \frac{G_n}{B_n}\text{: thus, 1/2, 1/4, 3/8, etc.}$$

$$\text{Females:}\quad AA = \frac{G_n \times G_{n+1}}{B_{2n-1}}\text{; thus 1/2, 3/8, 15/32, 55/128, etc.}$$

$$aa = \frac{G_n \times G_{n-1}}{B_{2n-1}}\text{; thus 0, 1/8, 3/32, 15/128, etc.}$$

$$Aa = 1 - AA - aa\text{; thus 1/2, 4/8, 14/32, 58/128, etc.}$$

Males and females equal in number.

(23) Sex-linked characters; a cross between AA and a−, then inbreeding, brother by sister. After *n* inbreedings:

$$\text{Males:}\quad A- = \frac{G_{n+1}}{B_n}. \qquad a- = \frac{G_n}{B_n}.$$

$$\text{Females:}\quad AA = \frac{I_{n+1}}{B_{n+1}}. \qquad Aa = \frac{F_{n+1}}{B_n}. \qquad aa = \frac{H_{n+1}}{B_{n+1}}.$$

(24) Sex-linked characters; cross between AA and a−, then inbreeding, daughters mated to fathers, sons to mothers (as could be carried out only in certain lower organisms). After *n* inbreedings:

$$\text{Males:}\quad A- = \frac{G_{2n+1}}{B_{2n}}\text{: thus, 3/4, 11/16, 43/64, etc.}$$

$$a- = \frac{G_{2n}}{B_{2n}}\text{: thus, 1/4, 5/16, 21/64, etc.}$$

$$\text{Females:}\quad AA = \frac{G_{2n+2} - 3^{n-1}}{B_{2n+1}};\ \text{thus } 4/8,\ 18/32,\ 76/128, \text{ etc.}$$

$$Aa = \frac{3^{n-1}}{B_{2n}};\ \text{thus } 1/4,\ 3/16,\ 9/64, \text{ etc.}$$

$$aa = \frac{G_{2n+1} - 3^{n-1}}{B_{2n+1}};\ \text{thus } 2/8,\ 8/32,\ 34/128, \text{ etc.}$$

Males and females equal in number.

In all these series the proportions of any given sort gradually approach a limit; thus in the last case (24) the limit for A− is 2/3; for a−, 1/3; for AA it is 2/3; for aa, 1/3; for Aa the limit is 0.

In the full paper there is a systematic presentation of formulae for the proportions in any generation, resulting from any of the main types of breeding, and with any of the common types of parental population, giving eighty-two numbered sets of formulae. In each case the limit approached is given, together with the number of generations of breeding required to come within 1% of that limit. The complete paper appears in the first number of *Genetics*.

ON THE EFFECTS OF FEEDING PITUITARY BODY (ANTERIOR LOBE) SUBSTANCE, AND CORPUS LUTEUM SUBSTANCE TO GROWING CHICKS

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In connection with the studies of Pearl and Surface[1] on the effect of pituitary substance on the function of egg production in the domestic fowl it seemed desirable to see whether the *initial* activation of the ovary could be accelerated by means of this substance. A pullet of a good producing strain hatched at the proper time and well grown will begin to lay when from five to six months of age ordinarily. Can such pullets be brought to sexual activity and laying any earlier if regularly fed pituitary substance?

Forty-five pure-bred Barred Plymouth Rock pullets, all hatched the same day (April 29, 1915) were divided into three lots of 15 each. They were so chosen that the total weights of the three lots, and thus the average weight per bird, were identical at the beginning of the experiment. Further great pains were taken to get birds of the same stage of maturity and physiological development so far as could be determined. Each bird in one lot (A) received *per os* 0.082 g. per day of pituitary body